

Explanation to Exercise 1, problem 1.3

The forward rates asked for are not the infinitesimal forward rates. We do not know the infinitesimal forward rates, because we know the zero rates only at discrete time points.

What I had in mind, was the one-year integrated forward rates $\int_{t-1}^t y_F(\tau) d\tau$. In other words, the accumulated force of interest that is consistent with the yield curve in each time interval $(t-1, t]$. They one-year integrated forward rates can be calculated from the yield curve as follows:

$$\int_0^t y_F(\tau) d\tau = y(t) \cdot t$$

$$\int_0^{t-1} y_F(\tau) d\tau = y(t-1) \cdot (t-1)$$

$$\text{Therefore } \int_{t-1}^t y_F(\tau) d\tau = y(t) \cdot t - y(t-1) \cdot (t-1) = y(t-1) + t \cdot (y(t) - y(t-1)).$$